Old Wine in New Bottles – Mass Spectra, Bootstrap and Hierarchy *

Y. Nambu

Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, IL 60637, USA

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A brief review is given of the problem of masses and mass spectra, and of an attempt to seek their dynamical origins.

1. Introduction

Among George Sudarshan's many-faceted contributions to physics, I am qualified to speak only to those in my own field: particle physics. Indeed, he supplied one of the key elements in the long chain of progress in our understanding of the weak forces when he, with Bob Marshak, was able to see through the utter confusion of experimental data, and to boldly propose, earlier than anybody else, the universal V–A structure of the weak interactions. While reading the memoirs they have written on the subject, I noticed that the first preprint version of their work was mailed out on his birthday, September 16, thirty-four years ago from today [1]. We have come a long way since those days; today, it seems fair to say that the theory of weak interactions is almost complete, but not quite yet. It is not complete because the origin of the masses of all particles and fields in the Standard Model is intrinsically tied to the postulated Higgs field, but one does not have a good understanding of the true nature of the Higgs field, nor of the mass spectrum of the fermions; the masses of fermions are mere phenomenological parameters in the Standard Model (SM). It is the purpose of my talk to address this question from my own perspective, which is based on the BCS mechanism of dynamical (or spontaneous) symmetry breaking (SSB).

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Reprint requests to Prof. Dr. Y. Nambu.

2. An SSB Overview

Let me start with a philosophical comment. The weak force is the least understood of the four forces. It appears that the weak interaction is intimately tied to the origin of masses, mass hierarchy, and the complexity of mass spectra. The complexity of mass spectra leads then to the complexity of the real world. In this connection it is appropriate to repeat my favorite quotation from Einstein on his equation of gravity [2]. He noted an unpleasant asymmetry between the left- and right-hand sides of his equation in elegance and beauty. In my opinion this is precisely due to the complexity of mass spectra and mass hierarchy in the material part of the world. Without intrinsic mass scales, the right hand side could be as elegant and beautiful as the left-hand side. It then suggests that the fundamental laws of nature would have no intrinsic mass scales, but that the masses somehow are dynamically induced by the nature of the initial and boundary conditions of the world.

An example to support the above point of view has occurred in an analogy between hadron physics and superconductivity:
In particle physics:

a) The chiral invariance of the weak and strong interactions broken by the masses of fermions and the Yukawa couplings;
b) The Goldberger-Treiman relation characterizing the properties of the pion.

In superconductivity:

c) Mass (gap) generation and the violation of charge conservation in the BCS theory;
d) The loss and subsequent recovery of a symmetry characterizing an SSB.

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Now the characteristics of a BCS mechanism, as I would like to define it for my purposes, are as follows [3]:

a) A short range attraction between fermions leading to a Copper pair condensate in the ground state of a medium.

b) Existence of fermionic and bosonic low energy excitations:

fermionic modes:

$$\chi \sim \alpha \psi_{up} + \beta \psi^{\dagger}_{dn} ,$$

bosonic modes:

$$\pi \sim \psi_{up} \psi_{dn} - \psi^{\dagger}_{dn} \psi^{\dagger}_{up} ,$$
$$\sigma \sim \psi_{up} \psi_{dn} + \psi^{\dagger}_{dn} \psi^{\dagger}_{up} .$$

c) One basic mass scale Δ and simple relations among the low energy modes:

$$m_{\sigma}, m_{\chi}, m_{\pi} \sim (2, 1, 0) \Delta .$$

d) One dimensionless induced coupling parameter f among these modes:

$$f = \Delta / v ,$$

where v is another momentum scale characteristic of the underlying dynamics.

e) The above characteristics then lead naturally to an effective Landau-Ginzburg (GL) transcription of the BCS mechanism with a restrictive choice of parameters.

f) Furthermore, the same restrictions imply a kind of supersymmetry called quasi-supersymmetry among the fermionic and bosonic modes, in such a way that the static part of the GL Hamiltonian can be factorized in terms of fermionic operators [4].

g) There is a possibility of tumbling [5], i.e., repetition of a chain of SSB's in a hierarchy of energy scales due to the induced attractive forces of a σ exchange between fermions.

h) There is also at least a theoretical possibility of bootstrap, as I will come back to later, where the above chain reduces to a self-repeating cycle.

As concrete examples of the BCS mechanism, I list the following:

1. Superconductivity
2. ^{3}He superfluidity
3. Nuclear pairing
4. QCD
5. The Standard Model

Some comments are in order:

a) The fermionic and bosonic low energy modes with the properties described above have been confirmed in the cases 1. and 2.

b) In nuclei, with their finite degrees of freedom and finite size, the notion of SSB has only an approximate validity. Some of the bosonic modes are spurious, and the mass relations do not apply. But the GL translation still gives meaningful results [6].

c) The generally accepted view that the (constituent) quark masses in QCD are generated in QCD by SSB does not yet have a direct experimental support such as a phase transition in quark-glucon plasma. Also the nature of the QCD interaction, with its confining features, is more complicated than the conventional one.

d) As for the Standard Model, it is an open question whether the Higgs field is elementary, in which case it would not come under the BCS category, or it is composite, and if so, whether it is a composite of the fermions in SM or of some new fermions as in the technicolor theories.

3. Tumbling

There are two examples of tumbling that I am aware of, which are represented schematically below.

1)

1st stage:	atoms	→ (QED)	→ crystals	→ phonons as Goldstone bosons
2nd stage:	electrons in metals	→ (phonons)	→ super-conductors	→ fermionic and bosonic modes

2)

1st stage:	massless	→ (QCD)	→ massive quarks and baryons
		→ π, σ	and other mesons
2nd stage:	nucleons	→ (σ)	→ nuclei and nuclear pairing
			→ fermionic and bosonic modes in nuclei

Comments:

a) The first stage of the first example represents an SSB, but not of the BCS variety. The phonon is a π (Goldstone) mode, not a σ (Higgs) mode, which does not exist in this case.

b) In the second stage of the second example, the fermions and bosons in nuclei are those modes that correspond to the shell model states and collective modes in the sense of the Interacting Boson Model [6].

4. Bootstrap and the Standard Model

The bootstrap mechanism was originally proposed by Chew [7] in hadron physics. A typical example is the formation of the ϱ meson as a resonance of two π mesons in the s-channel due to the exchange of ϱ itself in the t-channel. Although it had only qualitative success in the original form, it inspired the concept of duality, which inspired the Veneziano model, which inspired the string model, which inspired the present superstring theory. There are no other concrete examples of the bootstrap mechanism, but the concept seems dynamically natural, and has an esthetic appeal. Basically it is a feedback mechanism, and there is no reason that this would not occur in other phenomena (for example, in strong coupling superconductors).

Recently I speculated [8] that the Higgs mechanism in the Standard Model might be of such a nature. If so, SM would be a dynamically closed system in the original sense of Chew's bootstrap, and the Higgs boson would be regarded as a composite object made of the fermions in SM. It makes sense now that the top quark, unlike all other fermions, seems to have a mass of the order of the electroweak scale, hence a large Yukawa coupling with the Higgs, field, implying a strong attractive interaction with itself. The Higgs should then effectively be a $t-\bar{t}$ bound state, and its mass should be roughly twice that of the top. Radiative corrections tend to reduce the ratio. A similar idea has been proposed also by Miransky et al. [9]. A more detailed analysis was undertaken by Bardeen et al. [10] starting from the Nambu-Iona-Lasinio model which is believed to represent an effective theory in the large N limit of QCD. One gets a family of solutions for m_t and m_H depending logarithmically on a cutoff parameter. The top mass m_t tends to be large, of the order of the standard electroweak scale ~ 250 GeV, and higher than the current experimental estimate of ~ 140 GeV. My own approach based on the original renormalizable SM was somewhat different, and contained some theoretical problems which have not been resolved yet. But the qualitative features are similar.

I close with some additional remarks:

1) The entire mass matrix

The original question of the entire mass matrix of the fermions in SM remains unanswered by the bootstrap mechanism. If the bootstrap mechanism is correct, there will be little difference from the SM predictions except for the specific values predicted for m_t and m_H. A most desirable outcome of the bootstrap would be to be able somehow to explain the entire mass matrix. One does not yet know how, but there have been some speculations, again inspired by the BCS theory:

a) The "democratic mass matrix" ansatz plus perturbations (Fritch [11], Kaus and Meshkov [12]). Like in the original Cooper ansatz, the mass operator in the flavor space is assumed to have the same matrix elements everywhere in some physically meaningful basis in the zeroth approximation, which results in the splitting of one eigenstate (i.e., the top quark) from the rest. The more interesting question of the deviations from democracy is not answered by the ansatz. I once toyed with the idea of a random matrix for them, arguing with myself that, if the large N expansion is accepted for the QCD of three colors, it might not be entirely meaningless to talk about a random mass matrix of three flavors.

b) Heavy Majorana neutrinos. Another often invoked idea is the seasaw mechanism [13], which is a way of inducing small masses by leakage from large mass states through self-energy diagrams. In this connection, the problem of neutrino masses has recently become a rich field of speculation. Thus there may exist right-handed neutrinos with large Majorana masses, which induce small masses for the (almost) left-handed neutrinos. It is possible that the heavy majorana mass is also of the order of m_t, so that the Higgs field as a bound state is more or less equally shared between the top and the right-handed neutrinos [14].

2) Beyond SM

If one goes beyond the SM of three flavors, there is rich room for interpreting the compositeness of the Higgs field. The technicolor theories [15] belong to this kind, but they have been suffering from various difficulties. A simpler possibility that has also been exploited is to introduce a massive fourth flavor which bears the burden of creating the Higgs field. But the question of the large top mass would then be left hanging.

There have been some renewed attempts to go to higher groups than the standard one, e.g. by adding another gauge group which is responsible for mass generation [16]. My remark here concerns the relation between the mass generated through a Dyson-

Schwinger equation and one generated through an effective four-fermion representation of the gauge interaction [17]. The former deals with the Weisskopf-Furry self-energy diagram which is logarithmically divergent, whereas the latter is quadratically divergent. As was shown by Weisskopf [18], however, the logarithmic self-energy may be regarded as the result of a cancellation of two quadratically divergent terms. To see this, set the external fermion momentum to zero and write the product of two propagators in the diagram as a difference of two single propagators:

$$1/[(k^2 - m_f^2)(k^2 - m_V^2)] = [1/(k^2 - m_f^2) - 1/(k^2 - m_V^2)]/(m_f^2 - m_V^2)\,.$$

Then each term will produce a quadratic divergence, like a tadpole diagram of a fermion or of a gauge field. Their signs are right for this interpretation in the gauge boson mass is larger than the fermion mass. It shows that the effective four-fermion interaction represents only a part of the picture.

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